Name: Fengyan Yang 杨凤妍

Major: Physics (boling class）

Internship Unit: University of Arizona

Supervisor : Zheshen Zhang

Contact info: yfyyxr@gmail.com

NANKAI UNIVERSITY 1919

PROJECT

THE UNIVERSITY OF ARIZONA Global

Develop a simulation platform for studying the formation of optical frequency combs in ring resonators

◆**KEY WORDS:**

Micro resonator ; frequency combs; Simulation ; Calculation; Comsol; Matlab

◆**ABSTRCT :**

The objective of this project is to develop a simulation platform for studying the formation of optical frequency combs in ring resonators. The project entails two complementary tasks: 1) calculating the dispersion profile in a given waveguide geometry; and 2) solving the nonlinear dynamics of Kerr frequency combs.

1. *Calculating the dispersion profiles for bended and micro-ring resonator*

Dispersion also manifests itself as a temporal effect, known as group velocity dispersion (GVD). GVD causes a short pulse of light to spread in the time domain because different frequency components travel at different group velocities. GVD is defined as

$$D_1 = -\frac{\lambda}{c}\left(\frac{d^2 n_{eff}}{d\lambda^2}\right) \quad (1)$$

where λ is the wavelength of the light and n_{eff} is the effective refractive index that has accounted for both the material and the waveguide-geometric contributions. For a waveguide mode with a propagation constant β dependent angular frequency $\omega(\beta)$ GVD is defined as:

$$D_2 = -\frac{2\pi c}{\lambda^2}\frac{d^2\beta}{d\omega^2} \quad (2)$$

Where λ is the wavelength in the vacuum.

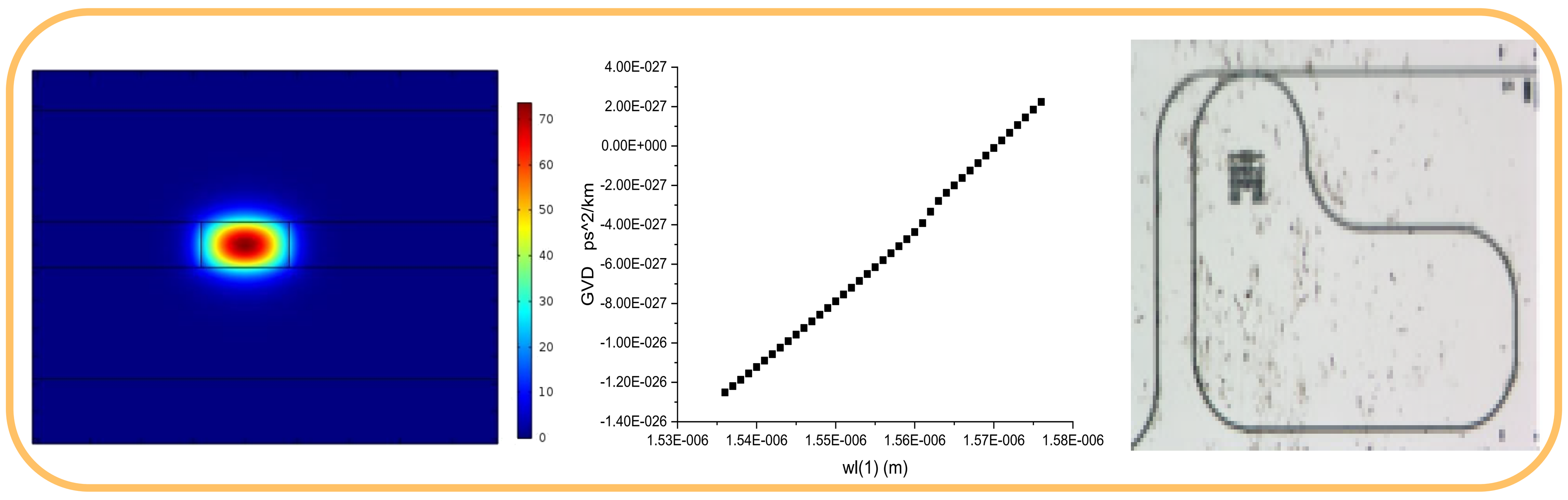


Figure 1. (a) Electric field distribution obtained by mode analysis. Waveguide parameters are taken from Ref. [10]. (b) Second-order dispersion profile. (c) Bird view of the waveguide used in simulation. The bending radius of the waveguide is large, so it is approximated as a straight waveguide. As such, the geometry is constructed in in "*2D*" instead of "*2D symmetric.*" in '2D symmetric', the simulation is conducting in a three -dimensional cylindrical coordinate. COMSOL can automatically form the whole geometry of a symmetric object by its cross section one draw in the graph window.)

2. *Solving frequency-combs dynamics in the frequency domain*

In the frequency domain, comb dynamics are described by a set of time-dependent coupling mode equations [1], which are a function of the main characteristics of the cavity, namely, the Kerr nonlinearity, absorption, coupling losses, and cavity dispersion including both the geometrical and material contribution.

$$\frac{dA_\eta}{dt} = -\frac{1}{2}\Delta\omega_\eta A_\eta + \frac{1}{2}\Delta\omega_\eta F_\eta e^{i(\Omega_0-\omega_\eta)t}\delta(l-l_0) - ig_0\sum_{\alpha\beta\mu} A_\alpha A_\beta^* A_\mu e^{i\omega_{\alpha\beta\mu\eta}t} \times \Lambda_l^{\alpha\beta\mu}\delta(l_a - l_\beta + l_\mu - l_\eta) \quad (3)$$

In order to solve the dynamics of the frequency combs in terms of the coupled mode equations, one needs to first convert the entire set of equations into MATLAB code. For example, to calculate frequency combs comprised of 201 modes, one has to work with 201 equations. A paramount step would be selecting the effective four-wave mixing modes and store the combinations of these coupling modes in a vector, whose element will later be used as the indices of the modes. After constructing the set of equations, one can use a fourth-order Runge-Kutta method to obtain the solution, with the Initial conditions specified by $\langle A_l(0)\rangle = 0$ and $\langle |A_l(0)|^2\rangle = \frac{1}{2}$

The frequency-domain method is very time consuming and therefore requires a high -performance computer. Fortunately, an accelerated numerical method based on the Fourier transform of the coupling term can be used .To show so, consider the coupled mode equation expressed as:

$$\frac{\partial A_\mu}{\partial\tilde{\tau}} = \left(-\frac{\kappa_\mu}{\kappa_0} + i\Omega_\mu\right)A_\mu + \delta_\mu f_0 + i\sum_{\alpha\beta\eta}\delta_{\mu-(\alpha-\beta+\eta)}A_\alpha A_\beta^* A_\eta \quad (4)$$

Using a tensor notation, one can convert the coupling term into:

$$\delta_{\mu,\alpha-\beta+\eta}A_\alpha A^*_\beta A_\eta = A_{\mu+\beta-\eta}A^*_\beta A_\eta = A_{\mu+k}A^*_{\eta+k}A_\eta \quad (5)$$

where $k=\beta-\eta$ .The right-hand side of this expression can be seen as two convolutions in the form of autocorrelations. As such, it can be derived by calculating the product in the conjugate domain followed by an inverse Fourier transform:

$$A_{\mu+k}A^*_{\eta+k}A_\eta = \mathcal{F}^{-1}[|a_j|^2 a_j] = \frac{1}{N}\sum_{j=0}^{N-1}(|a_j|^2 a_j)e^{(i2\pi j\mu)/N} \quad (6)$$

After doing the Fourier transform, the coupling part comprised of thousands of terms is reduce to a single term. Hence, the Fourier-transform technique substantially reduces the computational complexity.

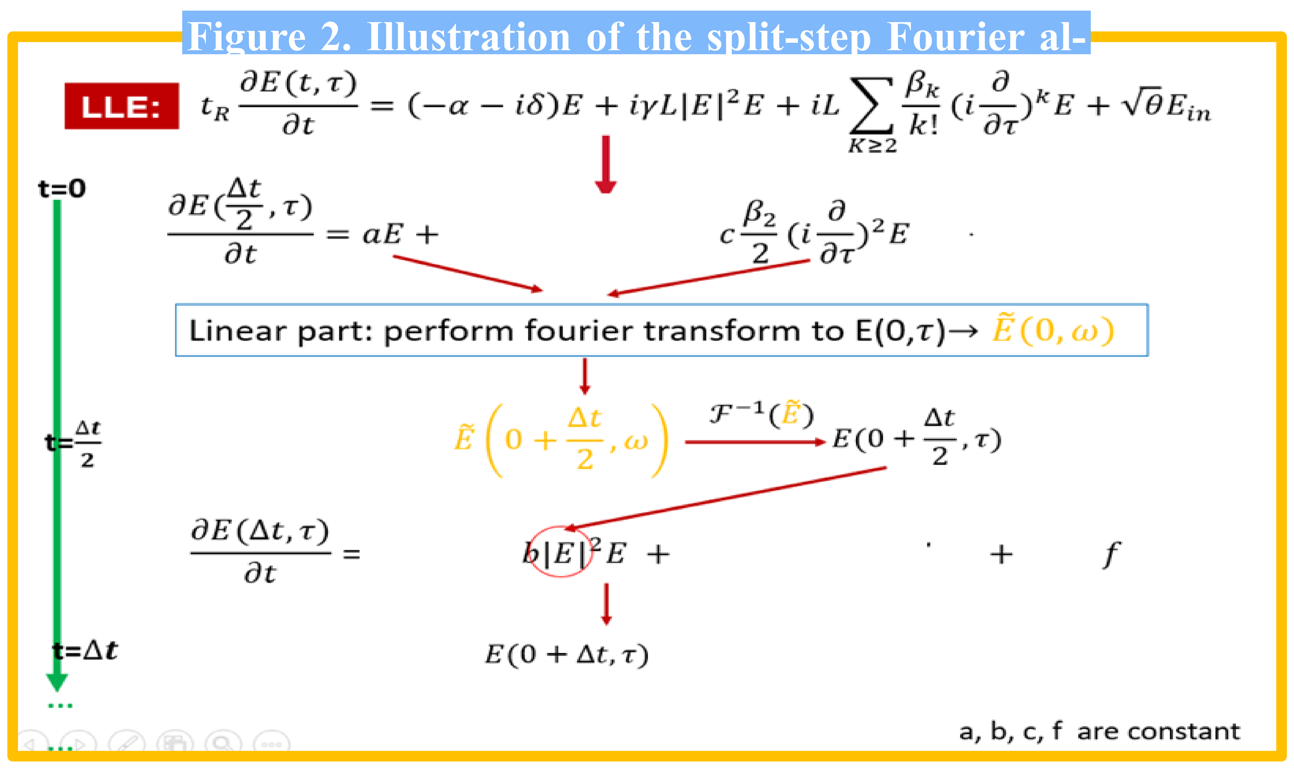
3. *The Lugiato-Lefever equation (LLE)*

The dynamic of Kerr frequency comb is governed by an externally driven and damped nonlinear Schrödinger (NLS) equation in the time-domain, known as the Lugiato-Lefever equation (LLE)[2].

$$t_R\frac{\partial E(t,\tau)}{\tau} = (-\alpha - i\delta)E + i\gamma L|E|^2E + iL\sum_{k\geq 2}\frac{\beta_k}{k!}\left(i\frac{\partial}{\partial\tau}\right)^k E + \sqrt{\theta}E_{in} \quad (7)$$

The general approach to solve this equation, is called the split-step Fourier method, is more commonly used, as it provides insights into the dynamics of frequency-combs formation.

Figure 2. Illustration of the split-step Fourier al-



4. *Cavity soliton*

Cavity solitons are potential information carriers in all-optical memories. The formation of cavity solitons is tightly associated with pump power and frequency. From an intuitive perspective, the formation of a single soliton requires the balance between dispersion and Kerr nonlinearity, as well as the balance between the four-wave mixing gain and loss.

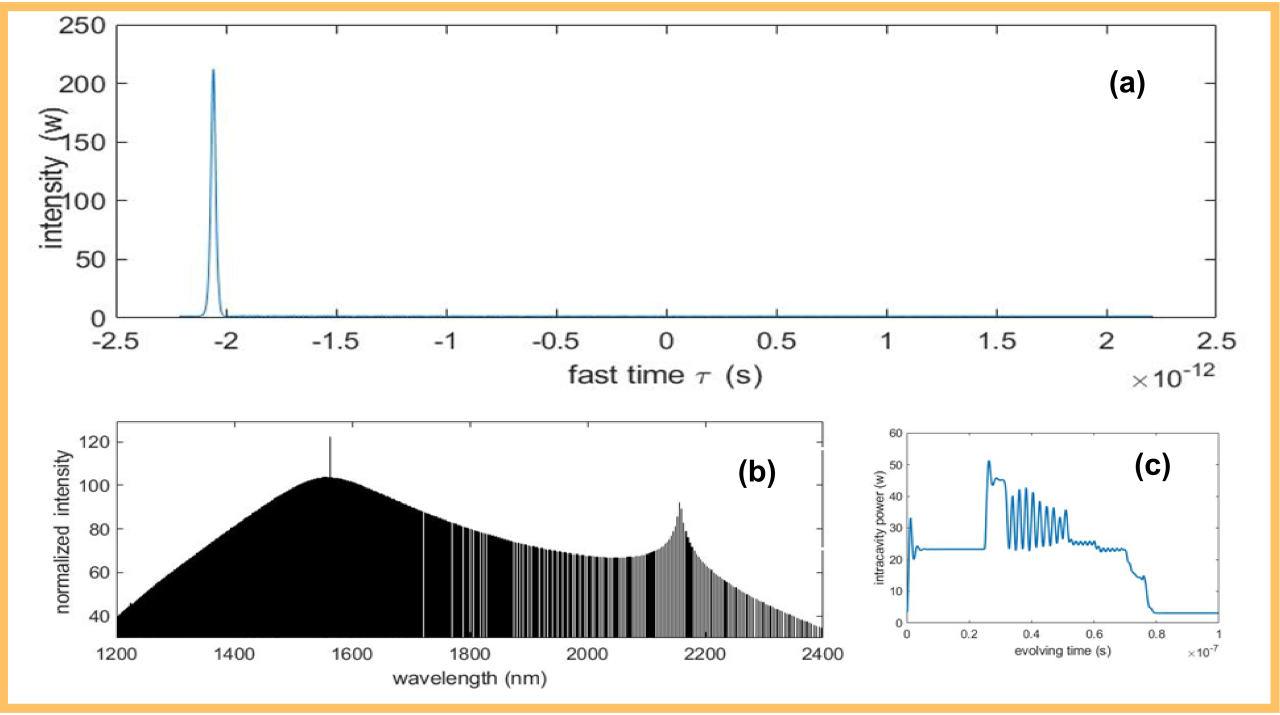


Figure 3. (a) Time domain intracavity field, which is a soliton. Cavity parameters are taken from Ref. [3] (b) Frequency comb spectrum. It is smooth and has a DW at 2156 nm. (c) intracavity power changes with evolving time.

5. *Conclusion*

During my stay in University of Arizona, I successfully built the simulation platform for micro-resonator based optical frequency combs, including using Comsol to calculate dispersion profile and using MATLAB to solve comb dynamics in both frequency and time domain.

6. *Reference*

[1] Y. K. Chembo and N. Yu, Modal expansion approach to optical-frequency-comb generation with monolithic whispering-gallery-mode resonators, Phys. Rev. A **82**, 033801 (2010).

[2] S. Coen, H. G. Randle, T. Sylvestre, and M. Erkintalo, Modeling of octave-spanning Kerr frequency combs using a generalized mean-field Lugiato–Lefever model, Opt. Lett. **38**, 37 (2013).

[3] M. R. E. Lamont, Y. Okawachi, and A. L. Gaeta, Route to stabilized ultrabroadband microresonator-based frequency combs, Opt. Lett. **38**, 3478 (2013).

Acknowledgement : **This summer internship is a valuable experience for me. I am really appreciated of my supervisor Prof. Zhang, it was his patient guidance and helpful suggestions that made my internship have a happy ending. Besides, the financial support of both Boling Class and my parents is also appreciated. At last, I'd like to give my sincere thanks to those people who help with my life in Tuscon, which contributed to the establishment of connection between me and the brand new world , thus preventing me from being lonely and solitude.**